MATHEMATICAL MODELS OF MERGERS: CONDITIONS OF APPLICATION AND CONCLUSIONS

Abstract

For many companies the basic way of enlarging its revenues and profits are mergers and acquisitions. These transactions have huge influence on the functioning of markets; therefore it is very important to understand how firms' connections influence markets and industries, and how the market structure influences profitability of connections. The analysis of theoretical models of horizontal mergers can be helpful, therefore, the aim of this study is to present the merger models as well as show their usefulness in the analysis of functioning markets and industries. The models of horizontal mergers are based on numerous simplifying assumptions, however, the analysis of the models can certainly show factors which should be specially considered in detailed analyses of functioning markets or the profitability of transactions.

Key words: A2 - Economics Education and Teaching of Economics, G34 – Mergers; Acquisitions; Restructuring; Corporate Governance, L22 - Firm Organization and Market Structure

Introduction

It has been researched that mergers and acquisitions happen cyclically. The great waves of firms' connections appear from time to time (Alberciak, 2002). However, even when the activity of firms diminishes in the area of connections, there is still a great amount of transactions. Many firms are still looking for the opportunity to connect with other company. Hence, better understanding of these processes is very important because the connections of firms have great impact on the functioning of markets. The analysis and prognostication of market events would be incomplete and inappropriate without taking into account the transactions of mergers and acquisitions. Therefore, the theoretical models of horizontal mergers can be very helpful.

The aim of this study is to present merger models as well as show their usefulness in the analysis of functioning markets and industries. These models are based on the basic assumption of the neoclassical theory of firm - the goal of a firm is profit maximization. The models (mainly based on the Cournot model) were created to find the conditions of profitability of firms' mergers. Those conditions are: proper market share of joining companies, convexity or concavity of the demand curve, the change of assumptions of conduct (the shift from the Cournot model to the Stackelberg model), the change of organizational structure or the suitable managers' commitment in the connection process. The models can be helpful in the analysis of specific transactions. Theoretical models obviously contain many simplifying assumptions, therefore any evaluations cannot be understood literally, however, they show directions and factors which an analyst should take into account. Hence, a hypothesis can be made that the analysis of theoretical models of horizontal mergers enriches the knowledge on real economic processes and shows directions for economic analyses.
The analysis of theoretical models of horizontal mergers

The first model relating to horizontal integration is the model of Salant, Switzer and Reynolds. According to the model of Salant et al., mergers are profitable only when firms taking part in the merger have market share of over 80 percent. The next models demonstrate that mergers can be profitable even if fewer firms take part in the connection but the merger will generate some cost savings or the demand function has suitable properties. However, firms interact in many ways. Mutual relationships among firms and their rivals, owners, customers or suppliers can cause that there may be many reasons for which mergers are profitable.

There are models of exogenous and endogenous mergers. In the exogenous merger models there is a group of firms whose members compare the benefits of getting together to the benefits of staying separately. In the endogenous merger models all firms have the possibility of choice whether to merge or not and how to react to a merger depending on the price for their assets and the other firms’ behaviour.

The model of Salant, Switzer and Reynolds (SSR model)

The model of Salant, Switzer and Reynolds (SSR) is based on the Cournot’s approach, in which every firm chooses such output level which maximizes its profit, taking into account the outputs of other firms. Using the SSR model we can compare the market on which every firm operates independently with the market on which some number of enterprises merged, keeping the condition of Cournot’s equilibrium. Comparison of these two cases allows to check when the profit of joint firms is smaller from sum of the firm’s profits before merging. The firms that will take part in merger will be called insiders, and the firms that will continue to operate independently after the merger will be called outsiders.

Model assumptions:
- there are \( n \) identical firms in an industry,
- there are \( m + 1 \) insiders, where \( m \) is an integer: \( 0 < m \leq (n - 1) \),
- marginal costs are constant.

If the insiders’ joint profits before the merger are denoted as \( \Pi^\text{NC}(n, m) \), and the insiders’ joint profit after merger is denoted as \( \Pi^\text{C}(n, m) \), then the increase in joint profits that results if \( m+1 \) insiders in an industry of \( n \) firms collude

\[
g(n, m) = \Pi^\text{C}(n, m) - \Pi^\text{NC}(n, m)
\]

Assuming that the marginal costs are constant, equal to \( \alpha \) and a linear demand curve is formulated:

\[
P = \beta - \sum_{i=1}^{n} Q_i
\]

where:
- \( \beta \) – a parameter of demand function,
- \( P \) – the market price,
- \( Q_i \) – the output of each firm in the industry,

then formula \( g(n, m) \) after rearranging is

\[
g(n, m) = \left( \frac{\beta - \alpha}{n - m + 1} \right)^2 - (m + 1) \left( \frac{\beta - \alpha}{n + 1} \right)^2 = (\beta - \alpha)^2 \left[ (n - m + 1)^{-2} - (m + 1)(n + 1)^{-2} \right].
\]

Table 1 shows how many firms would have to join \( (m + 1) \), to make a merger profitable, if there are \( n \) firms on the market.

Table 1. Profitability of mergers in the model of Salant, Switzer and Reynolds

<table>
<thead>
<tr>
<th>( n )</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>15</th>
<th>30</th>
<th>100</th>
<th>140</th>
<th>1000</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m+1 )</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>13</td>
<td>26</td>
<td>92</td>
<td>130</td>
<td>970</td>
</tr>
<tr>
<td>((m+1)/n (%))</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>80</td>
<td>83</td>
<td>86</td>
<td>88</td>
<td>89</td>
<td>90</td>
<td>87</td>
<td>92</td>
<td>93</td>
<td>97</td>
<td></td>
</tr>
</tbody>
</table>

Source: (Gonzalez-Maestre, Lopez-Cunat, 2001, p. 1268).
On the basis of the profitability condition it can be established, that:

- several oligopolists’ mergers into monopoly are always profitable,
- several participants’ mergers will bring loss to the parties taking part in a merger unless the market share of merging firms is over 80 per cent (for example, if \( n = 4 \), then the merger of 3 firms will cause the fall of the joint profits),
- connections are always profitable for the firms not taking part in a merger, therefore the merger can cause social external benefits because the decrease in insiders’ profits as well as the increase in a price for consumers can be outweighed by an increase in the outsiders’ producer surplus.

The SSR model excludes the possibility that firms can act like a Cournot in deciding how much to produce and can always benefit from a merger. Following that, there are three alternatives: (Salant, Switzer, Reynolds, 1983):

- firms act like Cournot players and some horizontal mergers may cause losses,
- firms do not always act like Cournot players in deciding how much to produce,
- firms act like Cournot players and some mergers never occur.

The SSR model skips the problem of external benefits despite the fact that authors anticipate possible increase in outsiders’ producer surplus. Conclusions from the model concentrate on merger’s profitability, so they can help firms in deciding if it is proper to join with other market’s participants. It can be helpful in strategic analyses and it can be the first step to analyse a specific market.

The model of Perry and Porter

The model of Perry and Porter makes no simplifying assumption, which is the loss of productive capacity of incorporated firms (as is the case in the SSR model). In the model of Perry and Porter the cost structure is crucial. It is assumed that there is some necessary input in the production process, for example capital, whose total supply is fixed in the industry.

Firms own different quantity of capital but the quantity of capital in a whole industry is 1 (100 percent). The cost function is denoted \( C(x, s) \), where \( x \) is the amount of production, and \( s \) is the fraction of the industry's capital owned by the firm. Constant returns to scale are assumed in the model.

Two situations can be analysed within the frame of the model: oligopoly with a competitive fringe as well as oligopoly of small and large firms (Perry, Porter, 1985).

a) Oligopoly with a Competitive Fringe

There are \( n \) oligopolists in the industry and each of them owns a fraction \( s \) of the industry capital and a competitive fringe owns the remaining capital, which is \( 1 - ns \). Small firms must own enough capital to join and form a new oligopolist. The condition that has to be fulfilled is: \( s \leq 1 - s \cdot n \). The competitive fringe firms produce such a quantity of goods that price and marginal costs are equal.

The model contains a parameter \( \delta \) whose value depends on the competitive intensity. It shows the conjectured output response of other oligopolists to a unit change in their own output (Bresnahan, 1981). If \( \delta = -1 \), then the oligopolists behave competitively. If \( \delta = 0 \), then oligopolists behave as firms in the Cournot model, assuming that the others will not respond to output changes. And if \( \delta = (n - 1) \), then the \( n \) oligopolists act in collusion to maximize joint profits. The parameter allows to introduce the degree of rivalry independently of the market model.

The oligopoly equilibrium output level is given by

\[
P + (1 + \delta) \cdot (1 + V') \cdot \frac{X}{n} \cdot P = MC \left( \frac{X}{n}, s \right),
\]

where:

- \( \delta \) – a parameter of competitive intensity,
- \( P \) – the market price,
- \( V' \) – the output of the fringe firms,
- \( X \) – the output of the oligopolists.

Solving the equilibrium formula we can obtain the equilibrium output of the oligopolists and
the fringe

\[ X(n) = \frac{(a - d)sn}{e + b \cdot [1 + s(1 + \delta)]}, \]

\[ V(n) = \frac{(a - d)(1 - sn)[e + b(1 - sn + s(1 + \delta))] \cdot [e + b(1 - sn)]}{(e + b \cdot [1 + s(1 + \delta)])}. \]

The output produced by any single oligopolist, \( \frac{X(n)}{n} \), is independent of \( n \), for some values of costs and demand parameters \( a, b, d, e \) and \( \delta \), and depends on quantity of capital \( s \). Industry output is the sum of \( X(n) \) and \( V(n) \), so that

\[ Z(n) = \frac{(a - d) \cdot K(n)}{e + b \cdot K(n)}, \text{ where } K(n) = \frac{e + b(1 - sn)[1 + s(1 + \delta)]}{e + b[1 - sn + s(1 + \delta)]}. \]

The mechanism of creating a profitable merger is as follows: when the number of oligopolists increases, the output of the industry decreases and the price of the product rises. This happens because the competitive fringe becomes smaller, so the industry behaves less competitively. The expected increase in price encourages small firms to join. However, the price rise has to be sufficient to offset the lower output level and to make the joint firms' profit greater than the sum of profits of firms before connection.

Considering the first connection of small firms into an oligopolist, it turns out that the merger condition is fulfilled for all values of parameters. There is always an incentive to form a dominant firm from the fringe. A second merger will be profitable for all nonnegative values of \( \delta \). If oligopoly behaviour becomes more competitive after the merger, merger will be less profitable. The third and the following mergers of the competitive fringe will occur when the coefficient \( \delta \leq 1 \).

b) Oligopoly of Small and Large Firms

The second situation, which can be analysed under the same assumptions as "the oligopoly with a competitive fringe", is an oligopoly of small and large firms. The difference is that there is no competitive fringe and the industry consists of large and small oligopolists. The large firms own, as earlier, \( s \) of the capital stock but the small ones own only \( s/2 \) of the capital stock.

The equilibrium output of the the two groups of firms

\[ X(n) = \frac{(a - d) \cdot [b(1 + \delta) + 2e / s] \cdot n}{\Delta(n)}, \]

\[ V(n) = \frac{2(a - d) \cdot [b(1 + \delta) + e / s] \cdot (1 / s - n)}{\Delta(n)}, \]

where: \( \Delta(n) = [b(1 + \delta) + \frac{e}{s}] \cdot [b(1 + \delta) + 2 \cdot \frac{e + b}{s}] \cdot b^2 \cdot (1 + \delta) \cdot n \),

\( X \) – the output of the large firms,
\( V \) – the output of the small firms,
\( a, b \) – parameters of demand function,
\( d, e \) – parameters of costs functions,
\( \delta \) – a parameter of competitive intensity.

Industry output is the sum of \( X(n) \) and \( V(n) \), so that

\[ Z(n) = \frac{(a - d) \cdot [b(1 + \delta)(2 / s - n) + 2e / s^2]}{\Delta(n)}. \]

The task of that model is to check whether small oligopolists will join in a large firm. When the number of large oligopolists increases, the output of the industry decreases and the good's price rises. It happens so, because the amount of production of two small firms before merger is greater than amount of production after connection. The merger will be profitable when the increase in price offsets the lower output level. Then the joint firm's profit (the large oligopolist’s) will be greater than the sum of profits of two small firms before connection.
Taking into consideration cost and demand functions assumptions, there are three cases, depending on values of the parameters of both functions:
- mergers of small oligopolists in a large firm are always profitable,
- mergers are never profitable,
- first mergers can be unprofitable, but when there will be sufficient number of large firms, then mergers of small oligopolists will be profitable.

The above-mentioned models show that in the dominant oligopoly model one or two new oligopolists can emerge or the whole competitive fringe can disappear, joining in a new oligopolist. In the oligopoly of Large and Small Firms model we could find no mergers at all or it can be found that the mergers will not occur unless the industry is sufficiently concentrated to begin with.

**The model of Fauli-Oller**

The model of Fauli-Oller develops the results reached by Salant, Switzer and Reynolds as well as by Cheung (Cheung, 1992). The Salant et al. showed that mergers are unprofitable if joining firms do not possess at least 80 percent of market share. Cheung showed that if the marginal revenue of industry decreases, the 50 percent market share is enough to make the merger profitable. That difference results from the assumption of the concavity of a market demand function. The greater the degree of concavity of demand, the lower the profitability of a merger.

In the model of Fauli-Oller the output of an industry \( X \) satisfies condition:

\[
P'(X) \cdot \frac{X}{n} + P(X) - c = 0,
\]

where:
- \( P(X) \) – the inverse demand function,
- \( c \) – the marginal cost,
- \( n \) – the number of independent firms.

The condition differentiated with respect to \( n \) is

\[
P'' X' \cdot \frac{X}{n} + \frac{P' X'}{n} - \frac{P' X}{n^2} + P' X' = 0,
\]

and after rearrangement

\[
X' = \frac{1}{n \cdot (\beta(n) + n + 1)},
\]

where:
- \( \beta \) - the degree of concavity of demand.

Individual profits of in equilibrium are given by:

\[
\pi(n) = \frac{[P(X(n)) - c]X(n)}{n},
\]

and after differentiating with respect to \( n \) we have

\[
\pi'(n) = P' X' \cdot \frac{X}{n} + X' \cdot \frac{P - c}{n} - \frac{(P - c)X}{n^2}.
\]

The proportional increase in profits due to a marginal increase in the number of firms is

\[
\frac{\pi''(n)}{\pi(n)} = \frac{X'}{X} \left( \frac{P' X}{P - c} + 1 \right) = \frac{1}{n}.
\]

Further rearrangement gives the merger profitability condition:

a merger will be profitable when the market share is

\[
f(N, b) = \frac{k(N, b) + 1}{N},
\]

when the condition \( \frac{P''(X) \cdot X}{P'(X)} = b \) is satisfied, and \( b \) is some constant degree of concavity, \( N \) is the number of firms competing according to the Cournot model.

The conclusion is: the less convex or more concave is the demand, the more difficult it is to
get a profitable merger. The merger will always be profitable, if the total market share of firms taking part in the connection is greater than certain threshold value of the coefficient of concavity of demand (Faulli-Oller, 1997).

The model of Leahy

The model defines conditions for mergers to be profitable for joining firms. Those conditions are based on the shape of reaction functions of firms acting like a Cournot player.

The main assumption of the model is that each firm produces one kind of good and the cost of launch of the second kind of product is so high that this is impossible without merger. When analysis is conducted under Cournot competition with strategic substitutes two cases appear:
- merger is unprofitable - this is the more probable, the more the outsiders’ production level increases, when the insiders’ production level decreases because of the merger,
- merger is profitable - if outsiders’ reaction function is sufficiently flat (due to convexity of demand felt by firms and the product differentiation) (Leahy, 2002).

The model of Leahy generalizes Faulli-Oller’s as well as the earlier authors’ conclusions. The obtained conclusions result from the shape of the reaction functions of firms not taking part in connection, influenced by different factors. Similarly like in Faulli-Oller’s model the more convex the demand, the easier it is to get a profitable merger. Leahy adds another factor, which makes it possible for mergers to be profitable - product differentiation. The less perfect the substitutes of the produced goods are, the higher the probability of the profitable merger.

The model of Higl and Welzel

The models presented above provide the analysis of the influence of a cost or demand function on merger profitability. In the model of Higl and Welzel the organizational structure of a company is considered. The model concerns an industry consisting of firms which in turn consist of several factories. The factories have some range of freedom in establishing the output and in this they have influence on output of the whole industry. The degree of centralization of a company is the key factor influencing its equilibrium and profitability of its merger (Higl, Welzel, 2005).

The factories establish their output quantities but, to some extent, they also take into account the guidelines of the headquarters. A firm rewards its factories in two ways. Firstly, they receive price for their products \( p_k \). Secondly, the factories have a share of the firm’s profit. The model assumes that all profits are distributed to the factories and the share of the profit is based on the output of a factory.

The necessary condition of profit maximisation with respect to the output of a factory is:

\[
1 - Y_k - \frac{dY_k}{dy_{ik}} y_{ik} - Y_k - \frac{dY_k}{dy_{ik}} y_{ik} - c = 0,
\]

where:
- \( Y_k \) – the total output of firm \( k \),
- \( y_{ik} \) – the quantity of production received by a firm from its factory \( i_k \),
- \( c \) – the marginal cost,
- \( Y_j \) – the sum of all competitors’ outputs.

Introducing parameter of co-ordination \( \gamma_k \), which expresses the change of firm \( k \)’s total output corresponding to a change in the output of factory \( i_k \): \( \gamma_k = \frac{dY_k}{dy_{ik}} = \sum_{j \neq k} \frac{dy_{jk}}{dy_{ik}} \), and using one of Cournot model assumption: \( \left( \frac{dY_k}{dy_k} = 0 \right) \), the maximisation condition can be written

\[
1 - Y_k - Y_k - \gamma_k y_{ik} - c = 0.
\]

The parameter \( \gamma_k \) can be changed into the parameter of co-ordination: \( \Gamma_k = \frac{\gamma_k}{n_k} \), and this parameter is
The parameter's value 1 means perfect vertical integration and value 0 means full autonomy of the production facilities (factories).

The analysis of industry's equilibrium leads to the following conclusions:

- if a merger leads to stronger centralization, then industry's output will decrease and the price will increase,
- if the parameter of co-ordination does not change, then neither the industry's output nor the price will change,
- if the new firm is more decentralized, then the welfare will increase (which means that the price of the product will decrease and the total output will increase).

In contrast to the SSR model results, according to the model of Higl and Welzel outsiders are not always better off. The merger is profitable for them only when it leads to centralization. When the merger leads to decentralization, the outsiders can suffer as a result, because the joint firm enlarges output after loosening its structure.

For insiders a merger can be profitable even without synergy effects. Choosing proper (de-)centralization coefficient is one of the conditions of success. It can also be measured how great should an insiders’ market share be so as not to have to change the degree of co-ordination after the merger.

General conclusion is that the smaller the joining companies’ market share, the more decentralized the decision-making should be. However, it is never profitable when factories behave as the perfect competitive. There should always be some co-ordination in a firm.

Mergers formed in order to strengthen market power are beneficial both for insiders and outsiders. The more mergers take into account the strategic effects of decentralization, the more insiders will benefit at the expense of outsiders. It is important as mergers are often undertaken not to strengthen the market power but to gain competitive advantage. The analysis shows that the internal organization has influence on economic effects of a merger. The optimal size of a firm is also dependent on its internal architecture.

The model of Banal-Estanol, Macho-Stadler and Seldeslachts

The model of Banal-Estanol, Macho-Stadler and Seldeslachts is a model of endogenous mergers. The models of endogenous mergers can provide a prediction of the final market structure. An industry structure is called stable when no firm wants to change the current configuration. The model of Banal-Estanol et al. widens the theory of horizontal mergers with interactions between concentration, managers' investments of skills and the influence of internal organization on the company’s efficiency (Banal-Estanol, Macho-Stadler, Seldeslachts, 2004). The model of endogenous mergers shows a market in which there are three firms, which is the minimum number to allow for mergers with "insiders" and "outsiders". The firms are supervised by "managers" or "management teams" - these terms are used as synonyms. Every manager has certain organizational and managerial competences which influence the costs of production. When firms merge some specific hard-to-trade assets integrate and this creates synergies. A merged firm cannot enjoy these synergies if it fires a manager since the fired manager would take his assets with him. This approach takes into account human capital. The presumed efficiency gains may not be realized because of two reasons:

- the right asset investment for the firm may imply a private cost for a manager (to avoid personal cost, a manager can reduce the probability of his firm being sold by reducing its attractiveness (Fulghieri, Rodrick, 2003)),
- building a common corporate goal out of two disparate cultures can be difficult and can even lead to less efficient and less profitable firms.

The model analyses two cases: when managers trust each other and they co-operate closely as well as when they do not trust each other and they do not co-operate, and investments are undertaken only when they are profitable for managers.

The model consists of a three-stage game:

- in the first stage, the managers choose whether to form a firm alone or join forces with other managers, determining the industry structure (Ω); three market structure can arise: monopoly, duopo-
Mathematical models of mergers

- in the second stage the production costs are determined; each manager decides to which extent he makes a costly investment to reduce production costs,
- in the third stage the formed firms compete according to the model of Cournot. The game is solved by backward induction. The solution of stage 3 is the equilibrium output and profit of firm \( w \)

\[
q_w = \frac{a - S - \sum_{v \in \Omega, v \neq w} I_v + r I_w}{r + 1},
\]

\[
\Pi_w = \left(1 - \sum_{v \in \Omega, v \neq w} I_v + r I_w \right)^2 \frac{(r + 1)^2}{(r + 1)^2}
\]

(under assumption: \( a - S = 1 \)),

where:
- \( a \) – a parameter of demand function,
- \( S \) – a parameter of marginal cost function,
- \( I \) – the investment of a firm, which lowers the marginal cost,
- \( r \) – the number of firms in an industry, after a merger in stage 1.

The model shows, among other things, the causes of failures of mergers. If managers decide to merge, anticipating no internal conflict and the conflict appears, then this inappropriate assessment of situation can make the merger unprofitable. Solutions of the model show that a monopoly can be beneficial when there is no conflict. If there is a conflict in the firm, the profits are higher when the market concentration is lower.

The considered model of endogenous mergers shows that internal problems can appear the moment managers decide to invest. The lack of mutual trust and inability to identify individual contributions may result in suboptimal decisions. People in a larger company have more incentives to invest because of synergies, but only do so when this is profitable. The problems of lack of trust are bigger in a larger firm because they can even offset the possible synergies thereby making the merged enterprise less efficient.

This model, widened with managerial factors, shows that the profitability of connections does not need to depend on a market or firm profile, but on managers’ decisions. In large companies managers’ decisions have often greater influence on firm’s activity than decisions of owners.

**The model of Rodrigues**

The model of Rodrigues is also a model of endogenous mergers. This model is based on a two-stage game. In the first stage companies decide whether to connect or not, and in the second stage the firms that remained on the product market compete with each other (Rodrigues, 2001).

The first stage is a sequential game in which firms announce one after another if they take part in a merger. Those firms that declared themselves available, merge into a single firm. The firms, that did not merge, remain independent competitors. In the second stage all firms compete with each other.

If the merger has no impact on firms’ conjectural variations \( \lambda \) and marginal costs \( c \), the output and the profit of firm \( i \) is equal to

\[
q_i^{pm} = \frac{a - c}{n - m + \lambda + 1},
\]

\[
\Pi_i^{pm} = \frac{\lambda + 1}{(n - m + \lambda + 1)^2} \cdot (a - c)^2 - \frac{m + 1}{\theta + 1},
\]

where:
- \( a \) – a parameter of demand function,
- \( \theta \) – a coefficient measuring the degree in which the merger allows cost savings,
$F$ – fixed costs,
n – the number of firms in an industry,
m + 1 – the number of firms that merge.

The result of the analysis of the model is a condition which can be used to examine the structure of a market in equilibrium depending on fixed costs and the coefficient of competitive intensity. At very low fixed costs only two structures of a market are possible - firms will merge into a monopolist if the competitive intensity is extremely high or the structure of the market will not change (Table 2). At very high fixed costs, firms will merge into a monopolist independently of competitive intensity. For intermediate values of fixed costs the equilibrium of market structure tends to stronger concentration as the competitive intensity increases.

The model shows that the firms’ propensity to merge is increasing with the growth of fixed costs and competitive intensity and decreasing with the growth of the number of companies in the industry (table 3).

Table 2. Number of firms in the equilibrium market structure, for an industry which initially had 10 firms, assuming various levels of competitive intensity ($\hat{\lambda}$) and fixed costs ($F/(a-c)2$)

<table>
<thead>
<tr>
<th>$\hat{\lambda}$</th>
<th>Fixed costs</th>
</tr>
</thead>
<tbody>
<tr>
<td>0,001</td>
<td>0,01</td>
</tr>
<tr>
<td>- 1,0</td>
<td>1</td>
</tr>
<tr>
<td>- 0,8</td>
<td>10</td>
</tr>
<tr>
<td>- 0,6</td>
<td>10</td>
</tr>
<tr>
<td>- 0,4</td>
<td>10</td>
</tr>
<tr>
<td>- 0,2</td>
<td>10</td>
</tr>
<tr>
<td>0,0</td>
<td>10</td>
</tr>
<tr>
<td>0,2</td>
<td>10</td>
</tr>
<tr>
<td>0,4</td>
<td>10</td>
</tr>
<tr>
<td>0,6</td>
<td>10</td>
</tr>
<tr>
<td>0,8</td>
<td>10</td>
</tr>
<tr>
<td>1,0</td>
<td>10</td>
</tr>
</tbody>
</table>


Table 3. Levels of competitive intensity ($\hat{\lambda}$) for various number of firms in an industry and fixed costs

<table>
<thead>
<tr>
<th>n</th>
<th>Fixed costs</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>[-0,384; 0,152]</td>
</tr>
<tr>
<td>4</td>
<td>[-0,595; -0,423]</td>
</tr>
<tr>
<td>5</td>
<td>[-0,691; -0,394]</td>
</tr>
<tr>
<td>10</td>
<td>[0,168; 0,703]</td>
</tr>
<tr>
<td>20</td>
<td>[-0,602; 0,532]</td>
</tr>
<tr>
<td>44</td>
<td></td>
</tr>
</tbody>
</table>


The presented models of exogenous and endogenous mergers show the influence of a market share, demand and cost functions' properties on stimulus to merge. Other factors which influence the mergers' profitability are: the degree of firms' decentralization, the degree of rivalry or strategic decisions, undertaken because of the fear that someone might take over their target-firm. The models of endogenous mergers develop the possibilities of models of exogenous mergers. The endogenous mergers models are helpful in finding the market equilibrium which shows the optimum number of firms...
depending on their maximum profit.

The way to use theoretical models in analysis of real merger and acquisition transactions

The theoretical models presented above can help in analysis of specific cases of mergers. Such analysis should consider of what importance the following factors are:

- the meaning of market power and the market share (the model of Salant et al., the model of Fauli-Oller, the model of Leahy),
- the degree of rivalry in an industry (the model of Perry and Porter, the model of Rodrigues),
- the market structure (the oligopoly of equivalent companies, the oligopoly of larger and smaller oligopolists, the oligopoly surrounded by a competitive fringe - the model of Perry and Porter),
- the specific market conditions (influencing the shape of demand or cost curves - the model of Fauli-Oller, the model of Leahy),
- the company's internal factors (the company's internal structure, the degree of workers' motivation, the joining companies' internal structure maladjustment - the model of Higl and Welzel, the model of Banal-Estanol et al.),
- the direction in which a market or industry goes (monopolisation of a market or staying competitive - the model of Rodrigues).

If we want to analyse some market and a specific connection, it is important to answer the above-mentioned questions first and then start the detailed analysis of the chosen aspects in the context of conclusions resulting from the models related to that problem. It should be remembered that theoretical models point only to the direction as well as factors which should be taken into account. Having that knowledge it is possible to analyse with methods which give detailed economic and financial conclusions on the profitability of a merger. Theoretical models have numerous simplifying assumptions; thanks to them models are conclusive, however, analysts should remember about their limitations. For example analysing the model of Salant et al. we reach the conclusion that an important aspect of investigation of profitability of connections is the matter of market share. However, it would be an incorrect conclusion, obviously that always and on every market we will not succeed if the parties of connection have less than 80 percent of market share. The success of a merger depends on some different factors, for example market power. Therefore it is necessary to answer all the above mentioned questions first, and then look over the theoretical models and their conclusions, to finally start the detailed analyses. Having the knowledge on the factors that should be particularly looked into, we can start the research in which four most important analytic areas are (Frąckowiak, 1998):

- the strategic analysis, providing the degree and the range of adjustment of market-production and economic-financial profiles of the potential buyer and the candidate to purchase, especially for mutual reduction of weaknesses and strengthening the strengths as well as for achieving the synergy effects,
- the analysis of the candidate's ability to generate income,
- the analysis of influence of the candidate's profitability potential on the price-earnings ratio after purchase,
- the analysis of the management of the firm-candidate to purchase.

Analyses conducted in the above listed areas should be accomplished regarding the conclusions from the earlier analysis of theoretical models. It will enrich the analyses of firms and markets and it will be more likely that any important factor will not be omitted.

Conclusion

The mathematical models of horizontal integration are a tool for analysing markets. The model of Perry and Porter considers two situations – oligopolists surrounded by many small firms as well as an industry consisting of smaller and larger oligopolists. Many markets are oligopolies; therefore, this model can be an important tool in studying them. The model in which a firm consists of smaller, independent units is the model of Higl and Welzel. Another strand in modeling merger formation are models of endogenous mergers. These models provide a prediction of the final market structure. It is also
interesting, regarding to real processes, to broaden the endogenous merger models with managerial factors, such as delegating the owner’s authority or the possibility of conflict in a firm. Hence the above mentioned models are tools that can be applied to analysing as well as prognosticating the development of markets.

**Comprehension check**

1. What are the two basic types of models of horizontal mergers?
2. What is the bottom limit of market share of joining firms for profitable merger in the model of Salant, Switzer and Reynolds? Do you think that in real transactions it is also the limit of profitability?
3. Which market model is the object of analysis of the model of Perry and Porter? Is this market model important in real-world analysis?
4. In which of the presented models the profitability of connections depends on the structure of joining firms? Present the conclusions from the analysis of the model.
5. What are the differences between two presented models of endogenous mergers? Which factors, according to your opinion, are more important - managerial factors or interaction between fixed costs and competitive intensity?

**Recommended readings**


REFERENCES:

7. Fulghieri F, Rodrick L., (2003), Synergies and Internal Agency Conflicts: The Double Edged Sword of Mergers, mimeo INSEAD.